14MAT11

USN

First Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting ONE full question from each module.***

Module – 1

1 a. Find the n^{th} derivative of $\dfrac{x^2}{(2x+1)(2x+3)}$. **(06 Marks)**

b. With the usual notations, prove that $\tan\phi = r\dfrac{d\theta}{dr}$ and find the angle between the radius vector and the tangent to the curve $r = a(1-\cos\theta)$ at the point $\theta = \dfrac{\pi}{3}$. **(07 Marks)**

c. Derive an expression to find the radius of curvature in polar form. **(07 Marks)**

2 a. If $y = \cos(m\log x)$, prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$. **(06 Marks)**

b. Find the pedal equation to the curve $\dfrac{2a}{r} = 1-\sin\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y} = \sqrt{a}$ at $\left(\dfrac{a}{4},\dfrac{a}{4}\right)$. **(07 Marks)**

Module – 2

3 a. Expand log(1+x) using Maclaurin's series upto the term containing x^4. **(07 Marks)**

b. State and prove Euler's theorem for homogeneous function of degree n. **(06 Marks)**

c. If $u = x+y+z$, $v = y+z$, $z = uvw$, find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. **(07 Marks)**

4 a. (i) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$. (ii) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x}{2}\right)^{\frac{1}{x}}$. **(06 Marks)**

b. If $u = \sin^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y} = 2\tan u$. **(07 Marks)**

c. If $u = f(x-y, y-z, z-x)$, then prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module – 3

5 a. A particle moves along the curve $\vec{r} = \cos 2t\hat{i} + \sin 2t\hat{j} + t\hat{k}$. Find the velocity and acceleration at $t = \dfrac{\pi}{8}$ along $\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}+\hat{k}$. **(06 Marks)**

b. Show that the vector, $\vec{F} = (3x^2-2yz)\hat{i} + (3y^2-2xz)\hat{j} + (3z^2-2xy)\hat{k}$ is irrotational and find ϕ such that $\vec{F} = \text{grad}\,\phi$. **(07 Marks)**

c. Use general rules to trace the curve $x^{2/3}+y^{2/3} = a^{2/3}$ **(07 Marks)**

6 a. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$, where $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$. **(06 Marks)**

b. Show that $\text{curl}(\text{grad}\,\phi) = 0$. **(07 Marks)**

c. By using the rule of differentiation under the integral sign, evaluate $\displaystyle\int_0^\infty \frac{e^{-x}\sin(\alpha x)}{x}dx$. **(07 Marks)**

Module – 4

7 a. Solve $\frac{dy}{dx}+\frac{y}{x}=y^2x$. **(06 Marks)**

b. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}}\cos^n x\,dx$. **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $y=x+Ce^{-x}$, where 'C' is the parameter. **(07 Marks)**

8 a. Evaluate $\int_0^1 x^5(1-x^2)^{\frac{5}{2}}dx$. **(06 Marks)**

b. Solve $(2x^3-xy^2-2y+3)dx-(x^2y+2x)dy=0$. **(07 Marks)**

c. A 12 volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ henry and resistance is 10 ohms. Determine the current if the initial current is zero. **(07 Marks)**

Module – 5

9 a. Solve the following system of equations: $x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$, by Gauss elimination method. **(06 Marks)**

b. Diagonalize the matrix $\begin{bmatrix}-1 & 1 & 2\\ 0 & -2 & -1\\ 0 & 0 & -3\end{bmatrix}$. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the matrix,

$$\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}.$$

Taking $[1\quad 0\quad 0]^T$ as the initial eigen vector carryout six iterations. **(07 Marks)**

10 a. Solve the following system by LU-decomposition method,
$x+y+z=1$, $3x+y-3z=5$, $x-2y-5z=10$ **(08 Marks)**

b. Find the inverse transformation of,

$y_1=4x_1+6x_2+6x_3$

$y_2=x_1+3x_2+2x_3$

$y_3=-x_1-4x_2-3x_3$ **(06 Marks)**

c. Reduce the quadratic form,
$3x^2+5y^2+3z^2-2yz+2zx-2xy$
to the canonical form. **(06 Marks)**

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